QUICK REFERENCE TO DISTRIBUTIONS USED IN FLOOD FREQUENCY ANALYSIS

Normal distribution

Cumulative distribution function:
\[
F(x) = \int_{-\infty}^{x} \frac{1}{b\sqrt{2\pi}} \exp\left\{-\frac{(x-a)^2}{2b^2}\right\} dx, \quad -\infty < x < \infty
\]

Cumulative distribution function of the standard normal variable Z:
\[
\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{z^2}{2}\right\} dz
\]

where:
\[
z = \frac{x-a}{b}
\]

Parameter estimation (method of moments and method of maximum likelihood):
\[
a = \bar{x}, \quad b = s_x
\]

Calculation of c.d.f. using tables:
\[
x \rightarrow z = \frac{x-a}{b} \quad \text{TABLE} \quad \Phi(z) = F(x)
\]

Calculation of c.d.f. in MS Excel:
\[
F = \text{NORMDIST}(x,a,b,\text{TRUE}) \quad \text{or} \quad F = \text{NORMSDIST}(z)
\]

Calculation of the inverse c.d.f. using tables:
\[
F \rightarrow \text{TABLE} \quad z \rightarrow x(F) = a + bz
\]

Calculation of the inverse c.d.f. in MS Excel:
\[
x = \text{NORMINV}(F,a,b) \quad \text{or} \quad x = a + b*NORMSINV(F)
\]
Lognormal distribution

Cumulative distribution function:

\[ F(x) = \int_0^x \frac{1}{xb\sqrt{2\pi}} \exp\left(-\frac{(\ln x - a)^2}{2b^2}\right) dx, \quad 0 < x < \infty \]

If random variable \( X \) is lognormally distributed, then the random variable:

\[ Y = \ln X \]

is normally distributed. With transformation

\[ Z = \frac{Y - a}{b} \]

lognormal distribution becomes the standard normal distribution.

Parameter estimation (method of moments):

\[ a = \ln \bar{x} - \frac{b^2}{2}, \quad b^2 = \ln(1 + \frac{C_v^2}{2}) \]

Parameter estimation (method of maximum likelihood):

\[ a = \bar{y}, \quad b = S_y \]

Calculation of c.d.f. using tables:

\[ x \rightarrow y = \ln x \rightarrow z = \frac{y - a}{b} \rightarrow \Phi(z) = F(x) \]

Calculation of c.d.f. in MS Excel:

\[ F = \text{LOGNORMDIST}(x,a,b) \quad \text{or} \quad F = \text{NORMSDIST}(z) \]

Calculation of the inverse c.d.f. using tables:

\[ F \rightarrow \text{TABLE} \rightarrow z \rightarrow y = a + bz \rightarrow x(F) = e^y \]

Calculation of the inverse c.d.f. in MS Excel:

\[ x = \text{LOGINV}(F,a,b) \quad \text{or} \quad x = \text{EXP}(a + b*\text{NORMSINV}(F)) \]

NOTE: It is possible to use base 10 logarithms instead of natural logarithms, i.e. to use the transformation \( Y = \log X \). Parameter estimates by the method of moments are then given with:

\[ a = \log \bar{x} - \frac{b^2}{2} \ln 10, \quad b^2 = \frac{\ln(1 + \frac{C_v^2}{2})}{\ln^2 10} \]

However, MS Excel functions \text{LOGNORMDIST} and \text{LOGINV} work with natural logarithms only. If base 10 logarithms are used, then calculations should be made with transformation \( Z = (\log X - a)/b \) and with functions for standard normal distributions \text{NORMSDIST} and \text{NORMSINV}. 
Pearson type 3 distribution

Cumulative distribution function:

\[
F(x) = \frac{1}{b \Gamma(a)} \left( \frac{x-c}{b} \right)^{a-1} \exp\left(-\frac{x-c}{b}\right) dx
\]

Parameter estimation (method of moments):

\[
a = \frac{4}{C_{sx}^2}, \quad b = \frac{S_x C_{sx}}{2}, \quad c = \bar{x} - ab
\]

Calculation using tables: Tables for the Pearson type 3 distribution provide relationship between frequency factor \(K\), values of c.d.f. and coefficient of skewness \(C_{sx}\). The frequency factor is defined with

\[
K(F) = \frac{x(F) - \bar{x}}{S_x}
\]

Note that there is no need to estimate parameters in order to make distribution calculations using tables.

Calculation of c.d.f. using tables:

\[
x \rightarrow K = \frac{x - \bar{x}}{S_x}, \quad C_{sx} \rightarrow \text{TABLE} \rightarrow F(x)
\]

Calculation of inverse c.d.f. using tables:

\[
F, \quad C_{sx} \rightarrow \text{TABLE} \rightarrow K(F) \rightarrow x(F) = \bar{x} + S_x \cdot K(F)
\]

Calculation in MS Excel: MS Excel features built-in functions for the two-parameter gamma distribution, \text{GAMMADIST}(x,a,b,cum) and \text{GAMMAINV}(F,a,b), whose application is limited to positive values of the skew coefficient \(C_{sx}\) and parameter \(b\). These functions should be used for calculation of the Pearson type 3 distribution in the following way.

Calculation of c.d.f. in MS Excel:
- for \(C_{sx} > 0\) and \(b > 0\):
  \[F = \text{GAMMADIST}((x-c)/b,a,1,\text{TRUE})\]
- for \(C_{sx} < 0\) and \(b < 0\):
  \[F = 1 - \text{GAMMADIST}((x-c)/b,a,1,\text{TRUE})\]

Calculation of inverse c.d.f. in MS Excel:
- for \(C_{sx} > 0\) and \(b > 0\):
  \[x = c + b \cdot \text{GAMMAINV}(F,a,1)\]
- for \(C_{sx} < 0\) and \(b < 0\):
  \[x = c + b \cdot \text{GAMMAINV}(1-F,a,1)\]
Log-Pearson type 3 distribution

The easiest way to apply the log-Pearson type 3 distribution is to make logarithmic transformation:

\[ Y = \ln X \quad \text{or} \quad Y = \log X \]

and to apply the Pearson type 3 distribution to the random variable \( Y \).

Gumbel (EV1) distribution

Cumulative distribution function:

\[ F(x) = \exp\left\{ -\exp\left[ -\frac{x-u}{\alpha} \right]\right\}, \quad -\infty < x < \infty \]

Inverse cumulative distribution function:

\[ x(F) = u + \alpha[-\ln(-\ln F)] \]

Standard Gumbel random variable is obtained with transformation:

\[ Y = \frac{X-u}{\alpha} \]

and the standard Gumbel distribution is:

\[ G(y) = \exp\{ -\exp[-y] \} \]

having inverse form:

\[ y(F) = -\ln(-\ln F) \]

Parameter estimation (method of moments):

\[ u = \bar{x} - 0.45S_x, \quad \alpha = 0.78S_x \]

General extreme value (GEV) distribution

Cumulative distribution function:

\[ F(x) = \exp\left\{ -\left[ 1 - k \frac{x-c}{\alpha} \right]^{-1/k} \right\}, \quad k \neq 0, \quad \text{for } k < 0: \quad x > c + \alpha/k \]

\[ \text{for } k > 0: \quad x < c + \alpha/k \]

Inverse cumulative distribution function:

\[ x(F) = c + \frac{\alpha}{k} [1 - (-\ln F)^k] \]

Parameter estimation (method of moments): shape parameter \( k \) is determined numerically from:

\[ f(k) = \text{sgn}(k) \frac{-\Gamma(1+3k) + 3\Gamma(1+k)\Gamma(1+2k) - 2\Gamma^3(1+k)}{[\Gamma(1+2k) - \Gamma^2(1+k)]^{3/2}} = C_{sx} \]

Parameters \( \alpha \) and \( c \) are then calculated from:
\[ \alpha = \text{sgn}(k) \cdot \frac{k \cdot S_x}{\left[ \Gamma(1 + 2k) - \Gamma^2(1 + 2k) \right]^{1/2}} \]

\[ c = \bar{x} - \frac{\alpha}{k} \left[ 1 - \Gamma(1 + k) \right] \]

Gamma function in MS Excel:
\[ \Gamma(x) = \text{EXP}(\text{GAMMALN}(x)) \]

**Exponential distribution**

Cumulative distribution function:
\[ F(x) = 1 - \exp \left( -\frac{x}{a} \right) \]

Inverse cumulative distribution function:
\[ x(F) = -a \ln(1 - F) \]

Parameter estimation:
\[ a = \bar{x} \]

Calculation of c.d.f. in MS Excel:
\[ F = \text{EXPONDIST}(x,a,\text{TRUE}) \]

**Two-parameter Weibull distribution**

Cumulative distribution function:
\[ F(x) = 1 - \exp \left\{ - \left( \frac{x}{a} \right)^b \right\} \]

Inverse cumulative distribution function:
\[ x = a \left[ -\ln(1 - F) \right]^{1/b} \]

Parameter estimation (method of moments): shape parameter \( b \) is determined numerically from:
\[ f(b) = \frac{\Gamma(1 + 2/b)}{\Gamma^2(1 + 1/b)} = 1 + C_r \]

and parameter \( a \) is then calculated from:
\[ a = \frac{\bar{x}}{\Gamma(1 + 1/b)} \]

Gamma function in MS Excel:
\[ \Gamma(x) = \text{EXP}(\text{GAMMALN}(x)) \]

Calculation of c.d.f. in MS Excel:
\[ F = \text{WEIBULL}(x,b,a,\text{TRUE}) \]