2.2.1.1.5  **RELATIONSHIP BETWEEN STRESSES AND DEFORMATION VELOCITY**

Now it shall be made clear, that the only way of making a connection between the stresses that act on a fluid element and the deformation velocity is by evaluating empirical physical experiments.

- We consequently only assume isotropic Newton fluids\(^2\).
- We further assume that the viscous stress tensor (Eq. 2-114) only depends on the deformation velocity tensor (Eq. 2-119), that means \(\tau_{ij}\) can be expressed as a function of the velocity gradients. The translation and the body rotation thus do not cause any surface forces.
- The functional relationship between \(\tau_{ij}\) and the velocity gradients is assumed to be linear and is independent from a rotation of the coordinate system and a change of the axis (isotropy\(^3\)).

With these preconditions we can derive the following relationship:

\[
\begin{align*}
\tau_{xx} &= \lambda \text{div} \bar{v} + 2\mu \frac{\partial u}{\partial x} \\
\tau_{yy} &= \lambda \text{div} \bar{v} + 2\mu \frac{\partial v}{\partial y} \\
\tau_{zz} &= \lambda \text{div} \bar{v} + 2\mu \frac{\partial w}{\partial z}
\end{align*}
\]

Eq. 2-122

and

\[
\begin{align*}
\tau_{xy} &= \tau_{yx} = \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \\
\tau_{zx} &= \tau_{xz} = \mu \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \\
\tau_{yz} &= \tau_{zy} = \mu \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)
\end{align*}
\]

Eq. 2-123

The factors of proportionality \(\mu\) and \(\lambda\) must have the same value for every direction because of the assumption of isotropy. If the equations are applied to simple flows like for example a Couette-flow, the equations are reduced to Newton’s friction law.

\[
\tau = \mu \frac{\partial u}{\partial y}
\]

Eq. 2-124

In this connection \(u\) is the flow velocity parallel to the wall and \(y\) is the coordinate normal to the wall. The proportionality constant \(\eta\) is described as dynamic viscosity. This way it can be seen that the factor \(\mu\) is equal to the viscosity of the fluid. The factor \(\lambda\) is only of importance if we look at compressible fluids, since \(\text{div} \bar{v}\) is zero for incompressible fluids.

---

\(^2\) A fluid whose shear stress \(\tau\) is proportional to the distortion- and shear speed, is a Newton's fluid (after Isaac Newton). Most of the fluids (e.g. water, air, lots of oils and gases) behave in this sense. Notwithstanding behave non-Newtonian fluids such as blood, glycerine or dough with a non-proportional, erratic flow behaviour. [wikipedia]

\(^3\) Isotropy (greek: isos = equally; greek: tropos = rotation, direction) is the independence of a property from the direction. For example, with a radiation isotropic is meant a radiation that is evenly emitted in all directions of the 3-dimensional space. [wikipedia]